

The Top Quark and other Fermion Masses

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ABSTRACT

Recent developments on approaches to the quark lepton mass problem are reviewed. In particular we discuss dynamical calculations of the top quark mass at (a) the infrared quasifixed point of the Minimal Supersymmetric Standard Model renormalisation group equations, and (b) a strongly first order critical point of the Standard Model effective potential. The phenomenology of symmetric mass matrix ansätze with texture zeros at the unification scale is also considered. The underlying chiral symmetry presumed responsible for the fermion mass hierarchy is discussed, with particular reference to superstring based models.

1 Introduction

The pattern of observed quark and lepton masses, their mixing and three generation structure form the major outstanding problem of particle physics. The fermion masses and mixing angles derive from Yukawa couplings, which are arbitrary parameters within the Standard Model (SM). Their experimental values provide our best clue to the physics of flavour beyond the SM. The main features requiring explanation are the following.

1. The large mass ratios between generations: $m_u \ll m_c \ll m_t$; $m_d \ll m_s \ll m_b$; $m_e \ll m_\mu \ll m_\tau$.
2. The large mass splitting within the third (heaviest) generation: $m_\tau \sim m_b \ll m_t$.
3. The smallness of the off-diagonal elements of the quark weak coupling matrix V_{CKM} .

The above features constitute the charged fermion mass and mixing hierarchy problem. The masses range over five orders of magnitude, from $\frac{1}{2}$ MeV for the electron to ~ 200 GeV for the top quark. It is only the top quark which has a mass of order the electroweak scale $\langle\phi_{WS}\rangle = 246$ GeV and a Yukawa coupling of order unity. All of the other fermion masses are suppressed relative to the natural scale of the SM. The main problem in understanding the fermion spectrum is not why the top quark is so heavy but rather why the electron is so light. Indeed, as the top quark mass is the dominant term in the fermion mass matrix, it is likely that its value will be understood dynamically before those of the other fermions.

As is well known, each generation (u,d,e, ν_e), (c,s, μ , ν_μ) and (t,b, τ , ν_τ) forms an anomaly free representation of the SM gauge group (SMG). The LEP measurements of the invisible partial decay width of the Z boson show that there are just three neutrinos of the usual type with masses less than $M_Z/2$ or more precisely [1]

$$N_\nu = 2.985 \pm 0.023 \pm 0.004 \quad (1)$$

This result is naturally interpreted to imply that there are just three generations of quarks and leptons, since neutrinos are purely left-handed and massless in the SM. It is of course possible to consider a fourth generation with a heavy neutrino at the electroweak scale [2] (e. g. by introducing one isosinglet right handed neutrino). It is also possible to add a fourth generation of quarks without accompanying leptons; in this case it is necessary to cancel the associated contributions to the gauge anomalies against those of some further particles. In fact the uncanceled quark anomalies make up a natural unit that is rather easily cancelled by a kind of techniquark. This can be done by adding an SU(5) component to the SMG together with a generation of SU(5) "quarks". The resulting model [3] has many features similar to the SM, including a natural generalisation of the SM charge quantisation rule. We do not consider these unconventional possibilities further here and just assume the usual picture of three SM generations.

As mentioned above there are no right-handed weak isosinglet neutrino states ν_R in the SM; so the Higgs mechanism cannot generate a neutrino mass term. However, in extensions of the SM, it is straightforward to generate

Majorana mass terms connecting the left-handed weak isodoublet neutrinos of the SM with the corresponding set of right-handed weak isodoublet anti-neutrinos. These Majorana mass terms break weak isospin by one unit ($\Delta t = 1$) as well as lepton flavour conservation. Such a $\Delta t = 1$ mass term can be generated by: (i) the exchange of the the usual Higgs tadpole $\langle\phi_{WS}\rangle$ twice, via a superheavy lepton L^0 intermediate state having the same gauge quantum numbers as ν_R (i. e. neutral) under the SM [4, 5]; or (ii) the exchange of a single weak isotriplet Higgs tadpole [6]. Method (i) has become known as the see-saw mechanism, since it generates a neutrino mass scale of $\langle\phi_{WS}\rangle^2/M_{L^0}$, suppressed by a factor of $\langle\phi_{WS}\rangle/M_{L^0}$ relative to the natural charged fermion mass scale of $\langle\phi_{WS}\rangle$. Some recent applications of the see-saw mechanism to make model predictions of neutrino masses and mixings will be found in other contributions to this meeting. Here we shall concentrate on the charged fermion mass problem.

Firstly, in section 2, we report on recent attempts to determine the top quark mass m_t (or more generally third generation masses) dynamically within the SM or the Minimal Supersymmetric Standard Model (MSSM). We then review some different approaches to the fermion mass and mixing hierarchy problem. In section 3 we consider ansätze for the fermion mass matrices. By imposing symmetries and texture zeros on the matrices, it is possible to obtain testable relations between the masses and mixing angles. (Texture zeros are small mass matrix elements which can be neglected to leading order.) In this approach the mass hierarchy is usually simply imposed, by fitting parameters in the ansätze to the data. It is natural to try to explain the hierarchical structure of these parameters in terms of (gauged) chiral flavour symmetries beyond those of the SM group. In section 4 we report on recent developments in models which generate realistic large mass ratios, via selection rules due to such new approximately conserved chiral flavour quantum numbers. The aim of such models is to reproduce all the fermion masses and mixing angles within factors of order unity. Section 5 contains our concluding remarks.

2 Dynamical Determination of the Top Quark Mass

We now consider some attempts to derive a fermion mass or mass relation from some dynamical or theoretical principle. Some time ago, Veltman [7] suggested that the fermionic and bosonic SM quadratic divergences should cancel, in order that the ultra-violet cut-off for the theory could be very high. It is, of course, one of the attractions of the MSSM that such a cancellation is automatic in supersymmetric (SUSY) theories. The Veltman condition for the cancellation of quadratic divergences at one loop in the SM gives the relation:

$$\sum_{leptons} m_l^2 + \sum_{quarks} m_q^2 = \frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{3}{4}m_H^2 \quad (2)$$

where the summation is over colour and flavour. It is not clear at what scale [8] this relation should be valid; but if applied at the electroweak scale it gives a SM Higgs particle mass of $m_H \simeq 330$ GeV.

More recently, in a toy model, Nambu [9] combined the Veltman condition for the cancellation of quadratic divergences with the idea that the vacuum energy density be minimised with respect to the Yukawa couplings, keeping all the other parameters fixed. This interesting idea of making the Yukawa couplings dynamical, constrained by the Veltman condition, naturally generates one Yukawa coupling (identified with the top quark) much larger than all the other ones [9, 10].

The Nambu model immediately raises the question of whether there is a physical justification for treating the Yukawa couplings as dynamical variables rather than numerical parameters. There is a possible answer to this question in superstring models, in which the Yukawa couplings depend on the vacuum expectation values (VEVs) of some gauge singlet scalar fields called moduli [10, 11, 12]. These moduli fields, T_α , which parameterise the size and shape of the six-dimensional compactified space, correspond to flat directions of the scalar potential in the effective four-dimensional supergravity theory. Thus their VEVs are determined by quantum corrections, with the possibility that some survive to the electroweak scale. In this case the low energy MSSM effective potential should be minimised with respect to these moduli, on which in turn the Yukawa couplings depend. In other words the effective potential should be minimised with respect to the Yukawa couplings, possibly subjected to constraints depending on the number of moduli fields remaining at low energy. This minimisation tends to drive the top quark mass to its largest allowed value and hence close to its infra-red quasi-fixed point value. Indeed, as we shall see, this MSSM fixed point prediction for the top quark is a common feature of many recent models of quark masses.

In fact we expect, on rather general grounds, that all coupling constants become dynamical in quantum gravity due to the non-local effects of baby universes [13]. As discussed in Holger Nielsen's talk here, we believe that the mild form of non-locality in quantum gravity, respecting reparameterisation invariance [14], leads to the realisation in Nature of the "multiple point criticality principle". According to this principle, Nature should choose coupling constant values such that the vacuum can exist in degenerate phases. In subsection 2.2 we apply this principle to the SM and obtain predictions for the top quark and Higgs boson masses [15], requiring the VEVs in two degenerate SM vacua to differ by an amount of order the Planck mass.

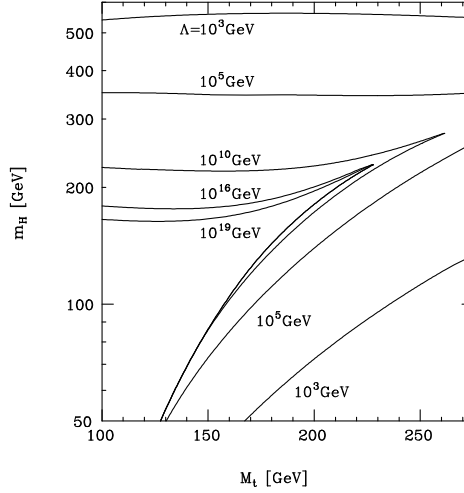


Figure 1: SM bounds in the (M_t, m_H) plane, for various values of Λ , the scale at which new physics enters.

It has been known for some time [16, 17] that the self-consistency of the pure SM up to some physical cut-off scale Λ imposes constraints on the top quark and Higgs boson masses. The first constraint is the so-called triviality bound: the running Higgs coupling constant $\lambda(\mu)$ should not develop a Landau pole for $\mu < \Lambda$. The second is the vacuum stability bound: the running Higgs coupling constant $\lambda(\mu)$ should not become negative leading to the instability of the usual SM vacuum. These bounds are illustrated [18] in figure 1, where the combined triviality and vacuum stability bounds for the SM are shown for different values of the high energy cut-off Λ . The allowed region is the area around the origin bounded by the co-ordinate axes and the solid curve labelled by the appropriate value of Λ . In the following we shall be interested in large cut-off scales $\Lambda \simeq 10^{15} - 10^{19}$ GeV, corresponding to the grand unified (GUT) or Planck scale. The upper part of each curve corresponds to the triviality bound. The lower part of each curve coincides with the vacuum stability bound and the point in the top right hand corner, where it meets the triviality bound curve, is the quasi-fixed infra-red fixed point for that value of Λ . The vacuum stability curve will be important for the discussion of the SM criticality prediction of the top quark and Higgs boson masses below. Before this however, we will discuss their quasi-fixed point values in the SM and the MSSM.

2.1 Infrared Fixed Point Predictions

The idea that some of the properties of the quark-lepton mass spectrum might be determined dynamically, as infrared fixed point values of the renormalisation group equations (RGEs) for the Yukawa coupling constants, was first considered [19] some time ago. It was pointed out that the three generation fermion mass hierarchy does not develop naturally out of the general structure of the RGEs. However it was soon realised [20] that the top quark mass might correspond to a fixed point value of the SM RGEs, predicting approximately $m_t \simeq 100$ GeV [20, 21]. In practice one finds that such an infrared fixed point behaviour of the running top quark Yukawa coupling constant $g_t(\mu)$ does not generically set in until $\mu < 1$ GeV, where the QCD coupling constant $g_3(\mu)$ varies rapidly. The scale relevant for the physical top quark mass prediction is of course $\mu = m_t$; at this scale $g_3(\mu)$ is slowly varying and there is an effective infrared stable quasifixed point (which would be an exact fixed point if $g_3(\mu)$ were constant) behaviour [22].

The quasifixed point prediction of the top quark mass is based on two assumptions: (a) the perturbative SM is valid up to some high (e. g. GUT or Planck) energy scale $\Lambda \simeq 10^{15} - 10^{19}$ GeV, and (b) the top Yukawa coupling constant is large at the high scale $g_t(\Lambda) \gtrsim 1$. Neglecting the lighter quark masses and mixings, which is a good approximation, the SM one loop RGE for the top quark Yukawa coupling $g_t(\mu) = \sqrt{2}m_t(\mu)/\langle\phi_{WS}\rangle$ is:

$$16\pi^2 \frac{dg_t}{d\ln\mu} = g_t \left(\frac{9}{2}g_t^2 - 8g_3^2 - \frac{9}{4}g_2^2 - \frac{17}{12}g_1^2 \right) \quad (3)$$

The gauge coupling constants $g_i(\mu)$ satisfy the RGEs:

$$16\pi^2 \frac{dg_i}{d\ln\mu} = b_i g_i^3; \quad \text{where} \quad b_1 = \frac{41}{6}, \quad b_2 = -\frac{19}{6}, \quad b_3 = -7 \quad \text{in the SM.} \quad (4)$$

The nonlinearity of the RGEs then strongly focuses $g_t(\mu)$ at the electroweak scale to its quasifixed point value. We note that while there is a rapid convergence to the top Yukawa coupling fixed point value from above, the approach from below is much more gradual. The RGE for the Higgs self-coupling $\lambda(\mu) = m_H^2(\mu)/\langle\phi_{WS}\rangle^2$

$$16\pi^2 \frac{d\lambda}{d\ln\mu} = 12\lambda^2 + 3(4g_t^2 - 3g_2^2 - g_1^2)\lambda + \frac{9}{4}g_2^4 + \frac{3}{2}g_2^2g_1^2 + \frac{3}{4}g_1^4 - 12g_t^4 \quad (5)$$

similarly focuses $\lambda(\mu)$ towards a quasifixed point value, leading to the SM fixed point predictions [22] for the running top quark and Higgs masses:

$$m_t \simeq 225 \text{ GeV} \quad m_H \simeq 250 \text{ GeV} \quad (6)$$

Unfortunately these predictions are inconsistent with the CDF and D0 results [23], which require a running top mass $m_t \simeq 170 \pm 12 \text{ GeV}$. Note that the running quark mass $m_q(\mu)$ is related to the physical or pole quark mass M_q , defined as the location of the pole in the quark propagator, by

$$M_q = m_t(M_q) (1 + 4\alpha_3(M_q)/3\pi) \quad (7)$$

at the one loop QCD level. The top quark pole mass is thus 5-6% larger than its running mass and we have taken [23] $M_t = 180 \pm 12 \text{ GeV}$. Consideration of higher order effects and the definition of the Higgs boson pole mass introduce many complicated issues [24, 25, 26, 27, 28] involving the SM renormalisation procedure.

There are two interesting modifications to the fixed point top mass prediction in the MSSM with supersymmetry breaking at the electroweak scale or TeV scale:

- The introduction of the supersymmetric partners of the SM particles in the RGE for the Yukawa and gauge coupling constants leads to a 15% reduction in the fixed point value of $g_t(m_t)$ [29, 30].
- There are two Higgs doublets in the MSSM and the ratio of Higgs vacuum values, $\tan\beta = v_2/v_1$, is a free parameter; the top quark couples to v_2 and so m_t is proportional to $v_2 = (174 \text{ GeV}) \sin\beta$.

The RGE for the top quark Yukawa coupling constant in the MSSM becomes

$$16\pi^2 \frac{dg_t}{d\ln\mu} = g_t \left(6g_t^2 + g_b^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{9}g_1^2 \right) \quad (8)$$

For $\tan\beta$ of order unity, the Yukawa couplings of the bottom quark $g_b(\mu)$ and the tau lepton $g_\tau(\mu)$ can still be neglected. The RGEs for the gauge coupling constants, eq. (4), are used with the supersymmetric values:

$$b_1 = 11, \quad b_2 = 1, \quad b_3 = -3 \quad (9)$$

The approach from above of $g_t(\mu)$ to its MSSM infrared fixed point value of approximately 1.1 is shown [31] in figure 2. The corresponding MSSM fixed point prediction for the top quark pole mass is [31]:

$$m_t(m_t) \simeq (200 \text{ GeV}) \sin\beta \quad (10)$$

which is remarkably close to the LEP and CDF results for $\tan\beta > 1$.

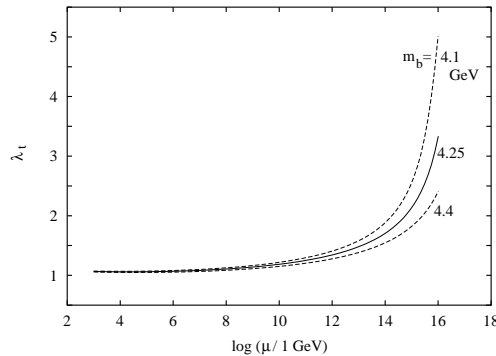


Figure 2: The rapid convergence from above of the top Yukawa coupling constant $\lambda_t \equiv g_t$ to the MSSM infra-red quasi-fixed point value as $\mu \rightarrow m_t$.

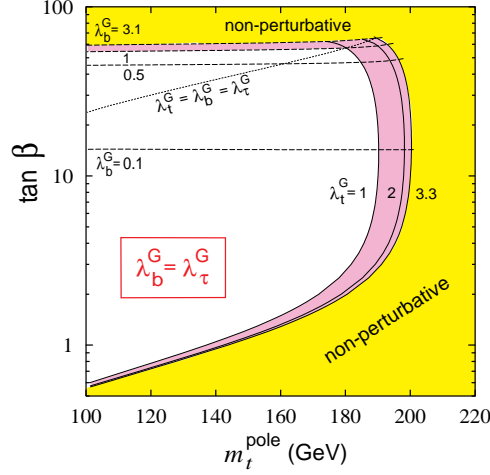


Figure 3: The top quark MSSM infrared fixed point region, corresponding to large values of its Yukawa coupling constant at the SUSY-GUT scale ($\lambda_t^G \equiv g_t^G \gtrsim 1$). Also shown are values of the bottom quark and tau lepton Yukawa couplings chosen to be equal at the SUSY-GUT scale.

The above quasifixed point value is of course also the upper bound on the top mass in the MSSM, assuming perturbation theory is valid in the desert up to the SUSY-GUT scale. It then follows that the experimental evidence for a large top mass requires $\tan\beta > 1$. We note that the minimal SU(5) SUSY-GUT symmetry relation between the bottom quark and tau lepton Yukawa coupling constants, $g_b(\Lambda) = g_\tau(\Lambda)$, is also only satisfied phenomenologically if the top quark Yukawa coupling is close to its infrared quasifixed point value, so that it contributes significantly to the running of $g_b(\mu)$ and reduces the predicted value of $m_b(m_b)$. In the SM the contribution of the top quark Yukawa coupling has the opposite sign and the SU(5) GUT prediction for $m_b(m_b)$ fails, as it is then phenomenologically too large.

The MSSM infrared fixed point value for the top quark pole mass is shown [31] in figure 3 as a function of $\tan\beta$. For large $\tan\beta$ it is possible to have a bottom quark Yukawa coupling satisfying $g_b(\Lambda) \gtrsim 1$ which then approaches an infrared quasifixed point and is no longer negligible in the RGE for $g_t(\mu)$. Indeed with $\tan\beta \simeq m_t(m_t)/m_b(m_t) \simeq 60$, we can trade the mystery of the top to bottom quark mass ratio for that of a hierarchy of vacuum expectation values, $v_2/v_1 \simeq m_t(m_t)/m_b(m_t)$, and have all the third generation Yukawa coupling constants large:

$$g_t(\Lambda) \gtrsim 1 \quad g_b(\Lambda) \gtrsim 1 \quad g_\tau(\Lambda) \gtrsim 1 \quad (11)$$

Then m_t , m_b and $R = m_b/m_\tau$ all approach infrared quasifixed point values compatible with experiment [32]. This large $\tan\beta$ scenario is consistent with the idea of Yukawa unification [33]:

$$g_t(\Lambda) = g_b(\Lambda) = g_\tau(\Lambda) = g_G \quad (12)$$

as occurs in the SO(10) SUSY-GUT model with the two MSSM Higgs doublets in a single **10** irreducible representation and $g_G \gtrsim 1$ ensures fixed point behaviour. However it should be noted that the equality in eq. (12) is not necessary to obtain the fixed point values. For example the principles of finiteness and the reduction of couplings [34] can be used to relate the Yukawa couplings to the SUSY-GUT coupling constant. In SU(5) finite unified theories the relationship is $g_t^2(M_X) = 4g_b^2(M_X)/3 = \mathcal{O}(1)$, giving a fixed point prediction similar to eq. (10). In fact one does not need a symmetry assumption at all, since the weaker assumption of large third generation Yukawa couplings, eq. (11), is sufficient for the fixed point dynamics to predict [32] the running masses $m_t \simeq 180$ GeV, $m_b \simeq 4.1$ GeV and $m_\tau \simeq 1.8$ GeV in the large $\tan\beta$ scenario. Also the lightest Higgs particle mass is predicted to be $m_{h^0} \simeq 120$ GeV (for a top squark mass of order 1 TeV). We also note that in superstring models tree level Yukawa couplings are proportional to the unified gauge coupling constant g_G , with a constant of proportionality either of order unity or exponentially suppressed. If the third generation Yukawa couplings have no residual moduli dependence at the electroweak scale, this superunification condition on the top quark Yukawa coupling $g_t(\Lambda)$ (also $g_b(\Lambda)$ and $g_\tau(\Lambda)$ for large $\tan\beta$) ensures that it is attracted towards its infrared fixed point value by its RGE anyway.

The origin of the large value of $\tan\beta$ is of course a puzzle, which must be solved before the large $\tan\beta$ scenario can be said to explain the large m_t/m_b ratio. It is possible to introduce approximate symmetries [35, 36] of the Higgs potential which ensure a hierarchy of vacuum expectation values - a Peccei-Quinn symmetry and a continuous \mathcal{R}

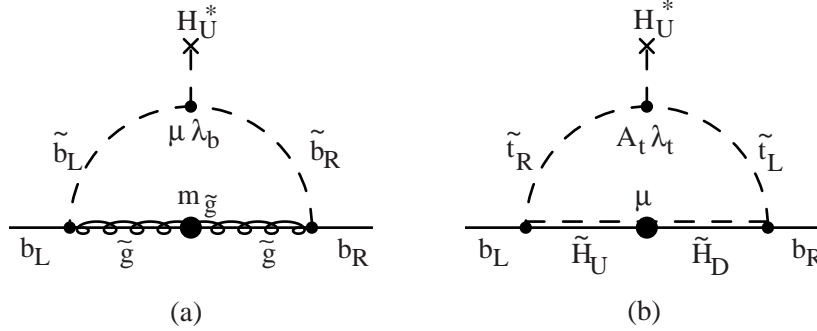


Figure 4: Leading one-loop MSSM contributions to the b quark mass.

symmetry have been used. The Peccei-Quinn symmetry essentially forbids the μ -term in the superpotential, while the \mathcal{R} symmetry requires the vanishing of gaugino masses $M_{1/2}$, of the SUSY-breaking trilinear scalar couplings A_i and of the bilinear SUSY-breaking Higgs coupling B . However these symmetries seem to be inconsistent with the popular scenario of universal soft SUSY breaking mass parameters at the unification scale and radiative electroweak symmetry breaking [37].

Also, in the large $\tan \beta$ scenario, SUSY radiative corrections to m_b are generically large: the bottom quark mass gets a contribution proportional to v_2 from some one-loop diagrams, whereas its tree level mass is proportional to $v_1 = v_2/\tan \beta$. The two dominant diagrams correspond to a bottom squark-gluino loop and a top squark-charged Higgsino loop respectively [35, 36], as shown in figure 4. Consequently these loop diagrams give a fractional correction $\delta m_b/m_b$ to the bottom quark mass proportional to $\tan \beta$ and generically of order unity [36, 37]. The presence of the above-mentioned Peccei-Quinn and \mathcal{R} symmetries and the associated hierarchical SUSY spectrum (with the third generation squarks and sleptons much heavier than the gauginos and Higgsinos) would protect m_b from large radiative corrections, by providing a suppression factor in the loop diagrams and giving $\delta m_b/m_b \ll 1$. However, in the absence of experimental information on the superpartner spectrum, precise predictions of the third generation quark-lepton masses in the large $\tan \beta$ scenario are, unfortunately, not possible.

2.2 Standard Model Criticality Predictions

Here we consider the idea [38], discussed in more detail by Holger Nielsen at this meeting, that Nature should choose coupling constant values such that several “phases” can coexist, in a very similar way to the stable coexistence of ice, water and vapour (in a thermos flask for example) in a mixture with fixed energy and number of molecules. The application of this so-called multiple point criticality principle to the determination of the top quark Yukawa coupling constant requires the SM (renormalisation group improved) effective Higgs potential to have coexisting vacua, which means degenerate minima:

$$V_{\text{eff}}(\phi_{\text{min } 1}) = V_{\text{eff}}(\phi_{\text{min } 2}) \quad (13)$$

This condition really means that the vacuum in which we live is barely stable; we just lie on the vacuum stability curve mentioned above and shown [27] in figure 5 for a cut-off $\Lambda = 10^{19}$ GeV. The important point for us, in the analogy of the ice, water and vapour system, is that the choice of the fixed extensive variables, such as energy, the number of moles and the volume, can very easily be such that a mixture must occur. In that case then the temperature and pressure (i. e. the intensive quantities) take very specific values, namely the values at the triple point, without any finetuning. We stress that this phenomenon of thus getting specific intensive quantities is only *likely* to happen for strongly first order phase transitions, for which the interval of values for the extensive variables that can only be realised as an inhomogeneous mixture of phases is rather large.

In the analogy considered here, the SM top quark Yukawa coupling and the Higgs self coupling correspond to intensive quantities like temperature and pressure. If these couplings are to be determined by the criticality condition, the two phases corresponding to the two effective Higgs field potential minima should have some “extensive quantity”, such as $\int d^4x |\phi(x)|^2$, deviating “strongly” from phase to phase. If, as we shall assume, Planck units reflect the fundamental physics it would be natural to interpret this strongly first order transition requirement to mean that, in Planck units, the extensive variable densities $\frac{\int d^4x |\phi(x)|^2}{\int d^4x} = \langle |\phi|^2 \rangle$ for the two vacua should differ by a quantity of order unity. Phenomenologically we know that for the vacuum 1 in which we live, $\langle |\phi|^2 \rangle_{\text{vacuum } 1} = (246 \text{ GeV})^2$ and

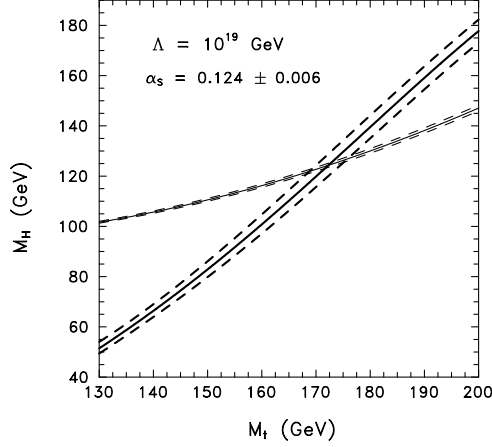


Figure 5: Diagonal (thick) lines: SM vacuum stability curve for $\Lambda = 10^{19}$ GeV and $\alpha_s = 0.124$ (solid line), $\alpha_s = 0.118$ (upper dashed line), $\alpha_s = 0.130$ (lower dashed line). Transverse (thin) lines: MSSM upper bounds on the Higgs pole mass M_H for a SUSY breaking scale of 1 TeV and α_s as in the diagonal lines.

thus we should really expect the Higgs field in the other phase just to be of Planck order of magnitude:

$$\langle |\phi|^2 \rangle_{\text{vacuum 2}} = \mathcal{O}(M_{\text{Planck}}^2) \quad (14)$$

This strong first order condition, eq. (14), and the degeneracy condition, eq. (13), are our two crucial assumptions.

In vacuum 2 the ϕ^4 term will a priori strongly dominate the ϕ^2 term. So we basically get the degeneracy to mean that, at the vacuum 2 minimum, the effective coefficient $\lambda(\phi_{\text{vacuum 2}})$ must be zero with high accuracy. At the same ϕ -value the derivative of the renormalisation group improved effective potential $V_{\text{eff}}(\phi)$ should be zero because it has a minimum there. Thus at the second minimum the beta-function β_λ vanishes as well as $\lambda(\phi)$, which gives to leading order, setting $\lambda = 0$ in eq. (5), the condition:

$$\frac{9}{4}g_2^4 + \frac{3}{2}g_2^2g_1^2 + \frac{3}{4}g_1^4 - 12g_t^4 = 0 \quad (15)$$

We use the renormalisation group to relate the couplings at the scale of vacuum 2, i. e. at $\mu = \phi_{\text{vacuum 2}}$, to their values at the scale of the masses themselves, or roughly at the electroweak scale $\mu \approx \langle \phi \rangle_{\text{vacuum 1}}$. Figure 6 shows the running $\lambda(\phi)$ as a function of $\log(\phi)$ computed for two values of $\phi_{\text{vacuum 2}}$ (where we impose the conditions $\beta_\lambda = \lambda = 0$) near the Planck scale $M_{\text{Planck}} \simeq 10^{19}$ GeV. Combining the uncertainty from the Planck scale only being known in order of magnitude and the QCD fine structure constant $\alpha_s(M_Z) = 0.117 \pm 0.006$ uncertainty with the calculational uncertainty [24, 25, 26, 27, 28], we get our predicted combination of top and Higgs pole masses:

$$M_t = 173 \pm 4 \text{ GeV} \quad M_H = 135 \pm 9 \text{ GeV}. \quad (16)$$

3 Mass Matrix Ansätze

The best known ansatz for the quark mass matrices is due to Fritzsch [39]:

$$M_U = \begin{pmatrix} 0 & C & 0 \\ C & 0 & B \\ 0 & B & A \end{pmatrix} \quad M_D = \begin{pmatrix} 0 & C' & 0 \\ C' & 0 & B' \\ 0 & B' & A' \end{pmatrix} \quad (17)$$

It contains 6 complex parameters A, B, C, A', B' and C', where it is necessary to *assume*:

$$|A| \gg |B| \gg |C|, \quad |A'| \gg |B'| \gg |C'| \quad (18)$$

in order to obtain a good fermion mass hierarchy. Four of the phases can be rotated away by redefining the phases of the quark fields, leaving just 8 real parameters (the magnitudes of A, B, C, A', B' and C' and two phases ϕ_1 and ϕ_2)

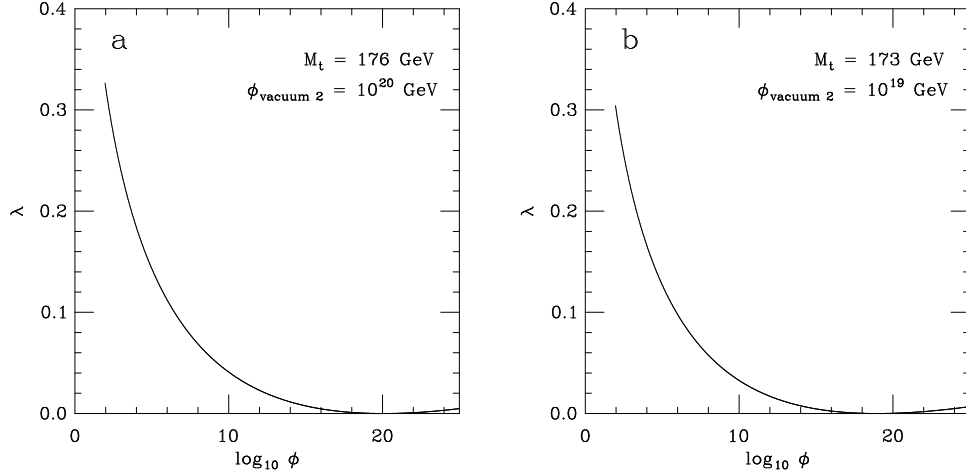


Figure 6: Plot of λ as a function of the scale of the Higgs field ϕ for degenerate vacua with the second Higgs VEV at the scale (a) $\phi_{\text{vacuum } 2} = 10^{20}$ GeV and (b) $\phi_{\text{vacuum } 2} = 10^{19}$ GeV. We formally apply the second order SM renormalisation group equations up to a scale of 10^{25} GeV.

to reproduce 6 quark masses and 4 angles parameterising V_{CKM} . There are thus two relationships predicted by the Fritzsch ansatz:

$$|V_{us}| \simeq \left| \sqrt{\frac{m_d}{m_s}} - e^{-i\phi_1} \sqrt{\frac{m_u}{m_c}} \right|, \quad |V_{cb}| \simeq \left| \sqrt{\frac{m_s}{m_b}} - e^{-i\phi_2} \sqrt{\frac{m_c}{m_t}} \right| \quad (19)$$

The first prediction is a generalised version of the relation $\theta_c \simeq \sqrt{\frac{m_d}{m_s}}$ for the Cabibbo angle, which originally motivated the ansatz and is well satisfied experimentally. However the second relationship cannot be satisfied with a heavy top quark mass $m_t > 100$ GeV [40] and the original Fritzsch ansatz is excluded by the data. Consistency with experiment can, for example, be restored by introducing a non-zero 22 mass matrix element [41], or by breaking the symmetry of the mass matrices and making the (2,3) and (3,2) elements of M_U unequal [42].

It is now common to make ansätze incorporating relationships between the fermion mass parameters at the GUT or the Planck scale. We have already mentioned the best known result: the simple SU(5) relation $m_b(\Lambda) = m_\tau(\Lambda)$ which is satisfied in SUSY-GUTs provided the top quark mass is near to its quasifixed point value [30, 43]. However the corresponding relations for the first two generations are not satisfied, as they predict for example

$$m_d/m_s = m_e/m_\mu \quad (20)$$

which fails phenomenologically by an order of magnitude. This led Georgi and Jarlskog [44, 45] to postulate the mass relations $m_b(M_X) = m_\tau(M_X)$, $m_s(M_X) = m_\mu(M_X)/3$ and $m_d(M_X) = 3m_e(M_X)$ at the GUT scale. Dimopoulos, Hall and Raby [30] revived these relations in the context of an SO(10) SUSY-GUT, combining the Fritzsch form for the up quark mass matrix $M_U = Y_u v_2$ with the Georgi-Jarlskog form for the down quark and charged lepton mass matrices $M_D = Y_d v_1$ and $M_L = Y_l v_1$:

$$Y_u = \begin{pmatrix} 0 & C & 0 \\ C & 0 & B \\ 0 & B & A \end{pmatrix} \quad Y_d = \begin{pmatrix} 0 & F e^{i\phi} & 0 \\ F e^{-i\phi} & E & 0 \\ 0 & 0 & D \end{pmatrix} \quad Y_l = \begin{pmatrix} 0 & F & 0 \\ F & -3E & 0 \\ 0 & 0 & D \end{pmatrix} \quad (21)$$

The phase freedom in the definition of the fermion fields has been used to make the parameters A, B, C, D, E and F real and we have again to assume: $|A| \gg |B| \gg |C|$ and $|D| \gg |E| \gg |F|$. Thus there are 7 free parameters in the Yukawa coupling ansatz and $\tan \beta$ available to fit 13 observables. Using the RGEs from the SUSY-GUT scale to the electroweak scale, this ansatz gives 5 predictions which, with the possible exception of V_{cb} discussed below, are in agreement with data for $1 < \tan \beta < 60$ [30, 46]. The simple SU(5) prediction, eq. (20), is replaced by the successful mass ratio prediction

$$(m_d/m_s)(1 - m_d/m_s)^{-2} = 9(m_e/m_\mu)(1 - m_e/m_\mu)^{-2} \quad (22)$$

Since the down quark matrix Y_d is diagonal in the two heaviest generations, one of the SUSY-GUT scale predictions is $V_{cb} \simeq \sqrt{\frac{m_c}{m_t}}$ [45]. Fits give m_t close to its fixed point and the large top Yukawa coupling causes V_{cb} to run between the

Solution	1	2	3	4	5
\mathbf{Y}_u	$\begin{pmatrix} 0 & C & 0 \\ C & B & 0 \\ 0 & 0 & A \end{pmatrix}$	$\begin{pmatrix} 0 & C & 0 \\ C & 0 & B \\ 0 & B & A \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & C \\ 0 & B & 0 \\ C & 0 & A \end{pmatrix}$	$\begin{pmatrix} 0 & C & 0 \\ C & B & B' \\ 0 & B' & A \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & C \\ 0 & B & B' \\ C & B' & A \end{pmatrix}$
\mathbf{Y}_d	$\begin{pmatrix} 0 & F & 0 \\ F^* & E & E' \\ 0 & E' & D \end{pmatrix}$	$\begin{pmatrix} 0 & F & 0 \\ F^* & E & E' \\ 0 & E'^* & D \end{pmatrix}$	$\begin{pmatrix} 0 & F & 0 \\ F^* & E & E' \\ 0 & E' & D \end{pmatrix}$	$\begin{pmatrix} 0 & F & 0 \\ F^* & E & 0 \\ 0 & 0 & D \end{pmatrix}$	$\begin{pmatrix} 0 & F & 0 \\ F^* & E & 0 \\ 0 & 0 & D \end{pmatrix}$

Table 1: Symmetric textures for the quark Yukawa matrices at the SUSY-GUT scale, which are consistent with the measured values of quark masses and mixing angles.

GUT and electroweak scales to a somewhat lower value. Nonetheless the fits still tend to make V_{cb} greater than the experimental value 0.040 ± 0.005 [1]. A fit satisfying Yukawa unification is obtained by setting $A = D$ and $\tan \beta \simeq 60$, for which $|V_{cb}| \geq 0.052$. This fit is of course subject to uncertainties due to the possibly large SUSY radiative corrections to m_b mentioned in the previous section and to other elements of the down quark mass matrix [47].

It is in fact possible to make a systematic analysis [48] of *symmetric* quark mass matrices with 5 or 6 “texture” zeros (counting pairs of off-diagonal zeros as one zero due to the symmetric structure assumed). The analysis is simply extended to include leptons by using the Georgi-Jarlskog ansatz: all the elements of Y_l are taken the same as for Y_d except the (2,2) element which is multiplied by the factor 3. The assumed symmetry of the Yukawa coupling matrices Y_u , Y_d and Y_l in generation space does not of course necessarily hold in the SM. However for unified models in which each generation appears as a single representation, such as SO(10), this symmetry is quite natural. Pragmatically this symmetry makes the analysis more tractable. It is also convenient to redefine the phases of the quark and lepton fields in such a way that the 3×3 symmetric Yukawa matrices are transformed into hermitean matrices. There are just 6 possible forms of symmetric Yukawa matrices with a hierarchy of three non-zero eigenvalues and the maximum number (three) of texture zeros. A survey of all six and five texture zero structures for the quark mass matrices, applied at the SUSY-GUT scale, reveals a total of 5 ansätze, each with five texture zeros, consistent with experiment. The corresponding quark Yukawa matrices are given in table 1. Note that solution 2 with $E' = 0$ reduces to the ansatz [30] of eq. (21). This ansatz with six texture zeros was excluded from table 1 since it gives large values for V_{cb} .

The parameters in the Yukawa matrices of the solutions in table 1 have a hierarchical structure similar to those in eq. (21). It is convenient [48] to reparameterise the Yukawa matrices in a way that keeps track of the order of magnitude of the various elements, as was done by Wolfenstein [49] for the quark mixing matrix in powers of the Cabibbo angle $\lambda \simeq 0.22$:

$$V_{CKM} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4) \quad (23)$$

For example solution 2 is well approximated by the following form:

$$Y_u = \begin{pmatrix} 0 & \lambda^6 & 0 \\ \lambda^6 & 0 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix} \quad Y_d = \begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 2\lambda^3 \\ 0 & 2\lambda^3 & 1 \end{pmatrix} \quad (24)$$

It is natural to interpret the small parameter λ as a symmetry breaking parameter for some approximate symmetry beyond those of the Standard Model Group (SMG). The nature of this symmetry is discussed in the next section.

The neutrino Majorana mass matrices generated by the see-saw mechanism in many extensions of the SM naturally have the above type of symmetric texture. Due to the hierarchical structure of their elements, there are two qualitatively different types of eigenstate that can arise [4]. In the first case, a neutrino can dominantly combine with its own antineutrino to form a Majorana particle. The second case occurs when a neutrino combines dominantly with an antineutrino, which is not the CP conjugate state, to form a 2-component massive neutrino. For example the electron neutrino might combine with the muon antineutrino. Such states naturally occur in pairs with order of

magnitude-wise degenerate masses. In the example given, the other member of the pair of Majorana states would be formed by combining the muon neutrino with the electron antineutrino. The hierarchical structure which gives rise to this second case is of course ruled out phenomenologically for the quark and charged lepton mass matrices, as none have a pair of states with order of magnitude-wise degenerate masses. However, considering two generations for simplicity, a neutrino mass matrix of the form

$$M_\nu = \begin{matrix} & \nu_1 & \nu_2 \\ \begin{matrix} \bar{\nu}_1 \\ \bar{\nu}_2 \end{matrix} & \begin{pmatrix} 0 & B \\ B & A \end{pmatrix} \end{matrix} \quad (25)$$

with the assumed hierarchy

$$|B| \gg |A| \quad (26)$$

could be phenomenologically relevant. The mass eigenvalues are $m_1 = B + A/2$ and $m_2 = B - A/2$, giving a neutrino mass squared difference $\Delta m^2 = 2AB$, and the neutrino mixing angle is $\theta \simeq \pi/4$ giving maximal mixing. Maximal neutrino mixing, $\sin^2 2\theta \simeq 1$, provides a candidate explanation for (i) the atmospheric muon neutrino deficit with $\Delta m^2 = 10^{-2} \text{eV}^2$ and $\nu_\mu - \nu_\tau$ oscillations, or (ii) the solar neutrino problem with $\Delta m^2 = 10^{-10} \text{eV}^2$ and $\nu_e - \nu_\mu$ vacuum oscillations.

4 The Fermion Mass Hierarchy and Chiral Symmetry

The fermion mass hierarchy can be expressed in terms of the Cabibbo angle $\lambda \simeq 0.22$ as a small expansion parameter by the order of magnitude values:

$$\frac{m_u}{m_t} = \mathcal{O}(\lambda^8), \quad \frac{m_c}{m_t} = \mathcal{O}(\lambda^4), \quad \frac{m_d}{m_b} = \mathcal{O}(\lambda^4), \quad \frac{m_s}{m_b} = \mathcal{O}(\lambda^2), \quad \frac{m_e}{m_\tau} = \mathcal{O}(\lambda^4 - \lambda^5), \quad \frac{m_\mu}{m_\tau} = \mathcal{O}(\lambda^2) \quad (27)$$

at the high energy, GUT or Planck, scale. It is natural to try to explain the occurrence of these large mass ratios in terms of selection rules due to approximate conservation laws. The mass m in the Dirac equation $i \not{\partial} \psi_L = m \psi_R$ essentially represents a transition amplitude between a left-handed fermion component ψ_L and its right-handed partner ψ_R . If ψ_L and ψ_R have different quantum numbers, i. e. belong to inequivalent irreducible representations (IRs) of a symmetry group G (G is then called a *chiral* symmetry), the mass term is forbidden in the limit of exact G symmetry and they represent two massless Weyl particles. G thus “protects” the fermion from gaining a mass. Note that this is exactly the situation for all the SM fermions, which are mass-protected by $SU(2)_L \times U(1)_Y$ (but not by $SU(3)_c$). The $SU(2)_L \times U(1)_Y$ symmetry is spontaneously broken and the SM fermions gain masses suppressed relative to the presumed fundamental (GUT or Planck) mass scale M by the symmetry breaking parameter:

$$\epsilon = \langle \phi_{WS} \rangle / M \quad (28)$$

The extreme smallness of this parameter ϵ constitutes, of course, the gauge hierarchy problem.

Here we are interested in the further suppression of the quark and lepton mass matrix elements relative to $\langle \phi_{WS} \rangle$. We take the view [19] that this hierarchy strongly suggests the existence of further approximately conserved chiral quantum numbers beyond those of the SMG broken, say, by terms of order $\mathcal{O}(\lambda)$. The SMG is then a low energy remnant of some larger group G and the fermion mass and mixing hierarchies are consequences of the spontaneous breaking of G to the SMG. The mass matrix element suppression factors depend on how the fermions behave *w.r.t.* G and on the symmetry breaking mechanism itself.

Consider, for example, an $SMG \times U(1)_f$ model, whose fundamental mass scale is M , broken to the SMG by the VEV of a scalar field ϕ_S where $\langle \phi_S \rangle < M$ and ϕ_S carries $U(1)_f$ charge $Q_f(\phi_S) = 1$. Suppose further that $Q_f(\phi_{WS}) = 0$, $Q_f(b_L) = 0$ and $Q_f(b_R) = 2$. Then it is natural to expect the generation of a b mass of order:

$$\left(\frac{\langle \phi_S \rangle}{M} \right)^2 \langle \phi_{WS} \rangle \quad (29)$$

via (see fig. 7) the exchange of two $\langle \phi_S \rangle$ tadpoles, in addition to the usual $\langle \phi_{WS} \rangle$ tadpole, through two appropriately charged vector-like superheavy (i.e. of mass M) fermion intermediate states [19]. Alternatively the transition can be mediated by a superheavy Higgs state having $Q_f = 2$. We identify $\epsilon_f = \langle \phi_S \rangle / M$ as the $U(1)_f$ flavour symmetry breaking parameter. In general we expect mass matrix elements of the form:

$$M(i, j) = \gamma_{ij} \epsilon_f^{n_{ij}} \langle \phi_{WS} \rangle, \quad \gamma_{ij} = \mathcal{O}(1), \quad n_{ij} = |Q_f(\psi_{L_i}) - Q_f(\psi_{R_j})| \quad (30)$$

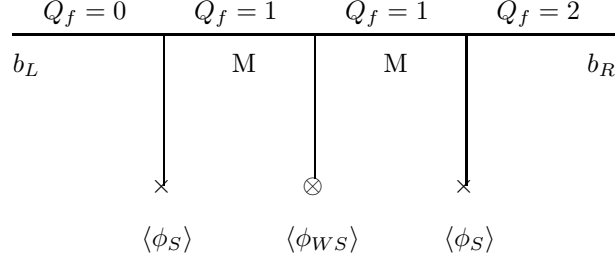


Figure 7: Feynman diagram which generates the b quark mass via superheavy intermediate states.

where n_{ij} is the degree of forbiddenness due to the $U(1)_f$ quantum number difference between the left- and right-handed fermion components. So the *effective* SM Yukawa couplings of the quarks and leptons to the Weinberg-Salam Higgs field

$$y_{ij} = \gamma_{ij} \epsilon_f^{n_{ij}} \quad (31)$$

can consequently be small even though all *fundamental* Yukawa couplings of the “true” underlying theory are of $\mathcal{O}(1)$. We are implicitly assuming here that there exists a superheavy spectrum of states which can mediate all of the symmetry breaking transitions; in particular we do not postulate the *absence* of appropriate superheavy states in order to obtain exact texture zeros in the mass matrices [50]. We now consider models based on this idea.

Recently a systematic analysis of fermion masses in $SO(10)$ SUSY-GUT models has been made [35, 51] in terms of effective operators obtained by integrating out the superheavy states, which are presumed to belong to vector-like $SO(10)$ $\mathbf{16} + \overline{\mathbf{16}}$ representations, in tree diagrams like fig. 7. The minimal number of effective operators contributing to mass matrices consistent with the low energy data is four, which leads to the consideration of GUT scale Yukawa coupling matrices satisfying Yukawa unification, eq. (12), and having the following texture

$$Y_i = \begin{pmatrix} 0 & z'_i C & 0 \\ z_i C & y_i E e^{i\phi} & x'_i B \\ 0 & x_i B & A \end{pmatrix} \quad (32)$$

where $i = u, d, l$. Here the x_i, x'_i, y_i, z_i and z'_i are $SO(10)$ Clebsch Gordon coefficients. These Clebschs can take on a very large number of discrete values, which are determined once the set of 4 effective operators (tree diagrams) is specified. A scan of millions of operators leads to just 9 solutions consistent with experiment, having Yukawa coupling matrices with a partial Georgi-Jarlskog structure of the form:

$$Y_u = \begin{pmatrix} 0 & \frac{-1}{27}C & 0 \\ \frac{-1}{27}C & 0 & x'_u B \\ 0 & x_u B & A \end{pmatrix} \quad Y_d = \begin{pmatrix} 0 & C & 0 \\ C & E e^{i\phi} & x'_d B \\ 0 & x_d B & A \end{pmatrix} \quad Y_l = \begin{pmatrix} 0 & C & 0 \\ C & 3E e^{i\phi} & x'_l B \\ 0 & x_l B & A \end{pmatrix} \quad (33)$$

For each of the 9 models the Clebschs x_i and x'_i have fixed values and the Yukawa matrices depend on 6 free parameters: A, B, C, E, ϕ and $\tan \beta$. The parameter hierarchy $A \gg B, E \gg C$ and the texture zeros are interpreted as due to an approximately conserved global $U(1)_f$ symmetry and the chosen superheavy fermion spectrum. The global $U(1)_f$ charges are assigned in such a way that only the 4 selected tree diagrams are allowed. In particular the texture zeros reflect the assumed absence of superheavy fermion states which could mediate the transition between the corresponding Weyl states. Each solution gives 8 predictions consistent with the data.

We now turn to models in which the chiral flavour charges are part of the extended gauge group. The values of the chiral charges are then strongly constrained by the anomaly conditions for the gauge theory. It will also be assumed that any superheavy state needed to mediate a symmetry breaking transition exists, so that the results are insensitive to the details of the superheavy spectrum. Consequently there will be no exact texture zeros but just highly suppressed elements given by expressions like eq. (30). The aim in these models is to reproduce all quark-lepton masses and mixing angles within a factor of 2 or 3.

The $SMG \times U(1)_f$ model obtained by extending the SM with a gauged abelian flavour group appears [52] unable to explain the fermion masses and mixings using an anomaly-free set of flavour charges. Models extending the SM (or the MSSM) with discrete gauge symmetries and having new interactions at energies as low as 1 TeV have also been investigated [53]; as have gauged finite non-abelian groups [54].

During the last year there has been considerable interest in models [55, 56, 57, 58, 59] extending the MSSM by a gauged abelian flavour group $U(1)_f$. The abelian symmetry $U(1)_f$ is usually (see however [60]) assumed to be spontaneously broken by

- (a) one chiral SMG-singlet field ϕ_S having a $U(1)_f$ charge $Q_f(\phi_S) = -1$, or
- (b) a vectorlike pair of SMG-singlet fields $(\phi_S, \bar{\phi}_S)$ with charges $Q_f = (-1, +1)$, which are assumed to acquire equal VEVs (along a D-flat direction).

The effective MSSM Yukawa couplings $y_{ij}^{u,d,l}$ break the $U(1)_f$ symmetry by

$$n_{ij}^{u,d,l} = Q_f(\psi_{L_i}) - Q_f(\psi_{R_j}) + Q_f(H_u, H_d, H_d) \quad (34)$$

units of $U(1)_f$ charge, where H_u and H_d are the two MSSM Higgs doublet fields. In case (a) the Yukawa coupling $y_{ij}^{u,d,l}$ will vanish whenever the excess charge $n_{ij}^{u,d,l}$ turns out to be negative, due to the holomorphy of the the superpotential which forbids the presence of the conjugate field ϕ_S^\dagger . For positive excess charge the value of $n_{ij} > 0$ (omitting the superscript u, d, l for convenience) gives the degree of suppression of the Yukawa coupling $y_{ij} \sim \epsilon_f^{n_{ij}}$ as in eq. (31). However it should be noted that the supersymmetric zeros (for $n_{ij} < 0$) may be filled in, by the wave function renormalisation of the kinetic terms described by the Kähler potential, up to at most a term of order $\epsilon_f^{|n_{ij}|}$. In case (b) the low energy Yukawa couplings will be of order $\epsilon_f^{|n_{ij}|}$ irrespective of the sign of the excess charge. Note also that if $Q_f(H) = Q_f(H_u) + Q_f(H_d) \neq 0$, the $\mu H_u H_d$ term is forbidden in the superpotential [58, 60]. Furthermore if $Q_f(H)$ is moderately negative, the μ -term can arise from the Kähler potential and naturally be of order the electroweak scale.

The new $U(1)_f$ gauge group is of course potentially anomalous and the conditions for the cancellation of anomalies provide strong constraints on the models. Due to the possible presence of other SMG-singlet fields, with non-zero $U(1)_f$ charges and no VEVs, it is assumed that the $U(1)_f^3$ gauge anomaly and the mixed $U(1)_f$ gravitational anomaly are cancelled against such spectator particles. However the conditions that the mixed $SU(3)^2 U(1)_f$, $SU(2)^2 U(1)_f$, $U(1)_Y^2 U(1)_f$ and $U(1)_f^2 U(1)_Y$ gauge anomalies should vanish have only been satisfied in one model [57] consistent with the phenomenological mass ratios eq. (27). This is a model of type (b), with a vectorlike pair $(\phi_S, \bar{\phi}_S)$, having the following $U(1)_f$ charge assignments (or an equivalent set) for the quarks and leptons:

$$\begin{pmatrix} d_L & u_R & d_R & e_L & e_R \\ s_L & c_R & s_R & \mu_L & \mu_R \\ b_L & t_R & b_R & \tau_L & \tau_R \end{pmatrix} = \begin{pmatrix} 2 & -2 & 0 & -5 & -7 \\ 1 & 1 & 1 & -3 & 5 \\ -1 & 3 & 1 & -6 & -4 \end{pmatrix} \quad (35)$$

The Higgs fields have $Q_f(H_u) = Q_f(H_d) = 4$ and the corresponding phenomenologically viable Yukawa matrices, generated using eq. (31) with $\gamma_{ij} \simeq 1$ and $\epsilon_f = \lambda$, have the form:

$$Y_u \simeq \begin{pmatrix} \lambda^8 & \lambda^5 & \lambda^3 \\ \lambda^9 & \lambda^4 & \lambda^2 \\ \lambda^{11} & \lambda^2 & 1 \end{pmatrix} \quad Y_d \simeq \lambda^2 \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^5 & \lambda^2 & \lambda^2 \\ \lambda^7 & 1 & 1 \end{pmatrix} \quad Y_l \simeq \lambda^2 \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^2 \\ \lambda^5 & \lambda^2 & \lambda^3 \\ \lambda^2 & \lambda^5 & 1 \end{pmatrix} \quad (36)$$

For this model the ratio of the two MSSM Higgs VEVs is given by $\tan \beta \simeq \lambda^2 m_t / m_b \simeq 2$.

It is also possible to generate phenomenologically viable Yukawa matrices with a single chiral ϕ_S field and appropriate $U(1)_f$ assignments. However in such type (a) models, the $SU(3)^2 U(1)_f$, $SU(2)^2 U(1)_f$ and $U(1)_Y^2 U(1)_f$ mixed anomalies no longer vanish [56]; they can only be cancelled by the Green-Schwarz mechanism [61], coming from an underlying string based model. String theories in 4 dimensions contain an axion-like field (the dilaton), which couples in a universal way to the divergence of the anomalous currents. By assigning a $U(1)_f$ gauge variation to the dilaton it is possible to cancel the anomalies, provided the mixed anomaly coefficients are in the same ratio as the $SU(3)$, $SU(2)$ and $U(1)_Y$ Kac-Moody levels k_i in the string theory. Gauge coupling unification at the string scale requires the Kac-Moody levels and the corresponding gauge coupling constants g_i to satisfy $k_3 g_3^2 = k_2 g_2^2 = k_1 g_1^2 = g_{\text{string}}^2$. The canonical normalisation of unified gauge coupling constants $g_3^2 = g_2^2 = \frac{5}{3} g_1^2$ (corresponding to the value $\sin^2 \theta_W = 3/8$ for the Weinberg angle) at the string scale requires the Kac-Moody levels to be in the ratio $k_3 : k_2 : k_1 = 1 : 1 : 5/3$. In order that the Green-Schwarz mechanism can function properly, the mixed anomaly coefficients have to be in the same ratio and it follows that [56, 58] $\det Y_d / \det Y_l \simeq \lambda^{Q_f(H)}$. The fermion mass ratios, eq. (27), then require $Q_f(H) = 0$ or -1 . The Green-Schwarz mechanism also requires [62] that the $U(1)_f$ symmetry is spontaneously broken an order of magnitude or so below the string scale.

In ref. [55], Ibanez and Ross considered type (b) models having symmetric mass matrices, with a view to constructing an anomaly free $MSSM \times U(1)_f$ model having the texture corresponding to one of the solutions in table 1. Different mass scales M and \bar{M} are used in the two MSSM Higgs sectors, giving two symmetry breaking parameters $\epsilon = \langle \phi_S \rangle / M$ and $\bar{\epsilon} = \langle \phi_S \rangle / \bar{M}$. They took the two MSSM Higgs doublets to be neutral under $U(1)_f$ and found a model similar to that of solution 2 as given in eq. (24). Again cancellation of the mixed anomalies of the $U(1)_f$ with the $SU(3)$, $SU(2)$ and $U(1)_Y$ gauge groups is only possible in the context of superstring theories via the Green-Schwartz mechanism with $\sin^2 \theta_W = 3/8$. The quarks and leptons are assigned the following $U(1)_f$ charges:

$$\begin{pmatrix} d_L & u_R & d_R & e_L & e_R \\ s_L & c_R & s_R & \mu_L & \mu_R \\ b_L & t_R & b_R & \tau_L & \tau_R \end{pmatrix} = \begin{pmatrix} -4 & 4 & 4 & -7/2 & 7/2 \\ 1 & -1 & -1 & 1/2 & -1/2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (37)$$

which generate Yukawa matrices of the following form [55]:

$$Y_u \simeq \begin{pmatrix} \epsilon^8 & \epsilon^3 & \epsilon^4 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^4 & \epsilon & 1 \end{pmatrix} \quad Y_d \simeq \begin{pmatrix} \bar{\epsilon}^8 & \bar{\epsilon}^3 & \bar{\epsilon}^4 \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & \bar{\epsilon} \\ \bar{\epsilon}^4 & \bar{\epsilon} & 1 \end{pmatrix} \quad Y_l \simeq \begin{pmatrix} \bar{\epsilon}^7 & \bar{\epsilon}^3 & 0 \\ \bar{\epsilon}^3 & \bar{\epsilon} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (38)$$

There is a Z_2 symmetry forcing the (1,3), (3,1), (2,3) and (3,2) elements of Y_l to vanish. Alternatively the lepton charges can be taken the same as the down quark sector giving $Y_l \simeq Y_d$. The correct order of magnitude for all the masses and mixing angles are obtained by fitting ϵ , $\bar{\epsilon}$ and $\tan \beta$. This is a large $\tan \beta \approx m_t/m_b$ model, but not necessarily having exact Yukawa unification.

The fermion mass and mixing angle predictions from these models could of course be more precisely tested if the $\mathcal{O}(1)$ coefficients γ_{ij} in eq. (31) were known. It has recently been suggested [63] that they could be determined as infrared fixed point values of the RGEs for the $MSSM \times U(1)_f$ model, supplemented by a large number of additional states in vector representations of the gauge group which acquire their mass one or two orders of magnitude below the string scale. In the presence of these extra states, the gauge couplings are no longer asymptotically free and the fundamental Yukawa couplings, being of $\mathcal{O}(1)$, evolve rapidly to their fixed point values just below the string scale. Furthermore these vectorlike states are assumed to form complete $SU(5)$ representations (even though the gauge group is not $SU(5)$), causing no relative evolution of the SMG gauge couplings to leading order and increasing the gauge unification scale close to the string scale. Encouraging results have been obtained.

As discussed by Dudas and Savoy at this meeting, the role of the $U(1)_f$ symmetry can be played by modular symmetry in effective superstring theory [12]. In this approach the small parameter ϵ is identified as the ratio of the real parts $Re T_\alpha$ of two moduli fields and the $U(1)_f$ charges are replaced by the modular weights of the fermion fields. This raises the possibility that the whole structure of the fermion mass matrices could be determined dynamically, by minimisation of the low energy effective potential with respect to the moduli fields. There have also been some recent developments on fermion masses in the antigrand unification model [64] as discussed in Nielsen's talk.

5 Conclusion

We described two promising candidate dynamical calculations of the top quark mass, using (a) the infrared quasifixed point value of the MSSM renormalisation group equations, and (b) the multiple point criticality principle in the pure Standard Model. In the large $\tan \beta$ scenario all the third generation masses are consistent with quasifixed point values and/or Yukawa unification. However the mystery of the large top to bottom quark mass ratio is then replaced by the mystery of a hierarchy of Higgs field VEVs; also the possibly large SUSY radiative corrections for large $\tan \beta$ make the mass predictions unreliable. There exist several symmetric mass matrix ansätze with texture zeros, giving typically 5 successful relations between mass and mixing parameters. Their hierarchical structure is most naturally explained in terms of an approximate chiral (gauge) symmetry beyond that of the Standard Model group. It is possible to generate phenomenologically viable fermion mass matrices in this way for the Minimal Supersymmetric Standard Model extended by a spontaneously broken abelian flavour symmetry group. In most cases this requires an underlying superstring model, in which the Green-Schwartz mechanism cancels anomalies with $\sin^2 \theta_W = 3/8$ at the string scale.

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